Journal of Sound and Vibration (1998) **218**(5), 825–844
Article No. sv981878

STRUCTURAL DAMAGE LOCALIZATION FROM MODAL STRAIN ENERGY CHANGE

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(*Received* 18 *November* 1996, *and in final form* 7 *July* 1998)

A method based on modal strain energy is presented for locating damage in a structure. This method makes use of the change of modal strain energy in each structural element before and after the occurrence of damage. Some properties of this Modal Strain Energy Change are given to illustrate its sensitivity in locating the structural damage. Information required in the identification are the measured mode shapes and elemental stiffness matrix only without knowledge of the complete stiffness and mass matrices of the structure. Several damage cases in a simulated structure are studied in which the effect of random error in terms of measurement noise in the mode shapes and systematic error in terms of errors from incomplete measurements are considered. This method is then applied to detect damage in a single-bay two-storey portal steel frame structure. Results illustrate that the MSEC is sensitive to damage, and the proposed method is simple and robust in locating single or multiple damages in a structure.

1. INTRODUCTION

Damages in the form of a loss of local stiffness in a structure would alter the system physical properties such as the vibration parameters of the structure, i.e., the modal frequencies, mode shapes and modal damping values. In such cases, the change in the vibration parameters can be used as indictors for damage detection. Techniques based on these changes for detecting damage in a structure have attracted much attention in recent years, and many approaches have been developed.

The majority of techniques used in the detection can be categorized into two groups. One group makes use of Finite Element Model (FEM) refinement algorithms. The damage detection problem is considered as a particular case of the general model updating problem where the aim of FEM refinement is to seek a refined model with its modal parameters in agreement with those obtained from the exepriment.

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These problems have been studied for several decades. An early work on Optimal Matrix Updates by Berman and Flannelly [1] presented the theory of incomplete models of dynamic structures and discussed the problem of calculating system matrices using incomplete test data. Several optimal matrix update methods were later proposed. Baruch and Itzhack [2] used the measured frequencies and mode shapes to modify a structural stiffness matrix based on the minimal Frobenius norm matrix adjustment technique. Berman and Nagy [3] adopted similar formulation to improve both the mass and stiffness matrices. Also Caesar [4] has given a very comprehensive review on the optimal matrix update approach.

Many strategies have been developed for locating the modelling errors in the FEM. These methods are also used for damage detection. Sidhu and Ewins [5] presented the Error Matrix Method in which changes in the stiffness and mass matrices between the analytical and experimental models were constructed and the modelling errors were located. Zhang and Lallement [6] proposed a complete procedure to localize the dominant errors and correct the selected parameters by a sensitivity method. Vibration control-based Eigenstructure Assignment Technique [7, 8] is another kind of model correction method. Zimmerman and Kaouk [9] applied this approach in a computationally attractive algorithm to identify the damage location, and the extent of damage was determined using a minimum rank updating theory [9, 10].

The other group of techniques to locate structural damage makes use of modal parameters as damage indicators. Cawley and Adams [11] used the ratio of the frequency changes in two modes as a damage indicator, which was proved to be a function of the damage location only. Locations indicated by the theoretically determined $\delta\omega_i/\delta\omega_j$ ratio equal to the exeprimentally measured values were possible damage sites. Lim and Kashangaki (1994) developed the best achievable eigenvectors which were computed based on a candidate set of assumed damage cases. The damage in the structure was located [12] by comparing the best achievable eigenvectors with the measured modes. Pandey and Biswas [13] used the change in flexibility matrix before and after the occurrence of damage in the structure as an indicator to locate the structural damage. Lin [14] multiplied the measured flexibility matrix by the undamaged analytical stiffness matrix. The resulting matrix should be an identity matrix if there is no damage. Inspecting rows and/or columns of the resulting matrix gives the location of the degrees of freedom (DOF) that are connected to the structural damage.

In the present work, the ratio of change in the Modal Strain Energy is proposed for detecting the damage location. This parameter is based on the estimation of the change of modal strain energy in each element after the occurrence of damage. Modal strain energy has been widely used to quantify the participation of each element in particular vibrating mode and in the selection of a candidate set of elements for damage localization [12]. Hearn and Testa [15] have illustrated that the ratio of the elemental strain energy to the total kinetic energy of the whole system is a fraction of the eigenvalue, and the ratio of this fraction for two different modes is dependent only on the location of the damage.

It will be demonstrated in the following sections that the proposed parameter is robust in locating the structural damage. Several simulated damage cases in the European Space Agency truss structure are used to examine the performance of this indicator and to illustrate the damage detection algorithm, and the experimental results of a single bay two-storey steel plane frame are studied. The effects of measurement noise and the mode shape expansion errors are discussed. Results indicate that the proposed method is effective and robust in locating single or multiple damage locations in the structure.

2. THEORY

The equations of motion for a n-DOF dynamic system can be expressed as

$$\mathbf{K\Phi} = \mathbf{M\Phi\Lambda}, \tag{1}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the $n \times n$ system stiffness and mass matrices, $\mathbf{\Phi}$ is the mode shape matrix, and $\mathbf{\Lambda}$ is the diagonal frequency matrix.

In actual structures damage may often affect the stiffness matrix but not the mass matrix of the system. In the theoretical development that follows, damage is assumed to cause a loss of stiffness in one or more elements of the system. The stiffness matrix, modal frequencies and mode shapes of the structure with damage can be represented by

$$\mathbf{K}^d = \mathbf{K} + \sum_{j=1}^{L} \Delta\mathbf{K}_j = \mathbf{K} + \sum_{j=1}^{L} \alpha_j \mathbf{K}_j \quad \text{with} \quad (-1 < \alpha_j < 0), \tag{2a}$$

$$\lambda^d = \lambda + \Delta\lambda, \tag{2b}$$

$$\mathbf{\Phi}_i^d = \mathbf{\Phi}_i + \Delta\mathbf{\Phi}_i = \mathbf{\Phi}_i + \sum_{j=1}^{m} c_{ij}\mathbf{\Phi}_j, \tag{2c}$$

where the superscript d denotes the damaged case, and α_j, and c_{ij} are coefficients defining a fractional change of the stiffness matrix and the mode shape vector; m is the number of modes considered and L is the total number of elements in the system.

2.1. DEFINITION OF THE LOCATION METHOD

The Modal Strain Energy (MSE) of the jth element and the ith mode before and after the occurrence of damage is defined as

$$MSE_{ij} = \mathbf{\Phi}_i^T \mathbf{K}_j \mathbf{\Phi}_i \quad \text{and} \quad MSE_{ij}^d = \mathbf{\Phi}_{di}^T \mathbf{K}_j \mathbf{\Phi}_{di}, \tag{3}$$

where MSE_{ij} and MSE_{ij}^d are respectively the undamaged and damaged MSE which are functions of the jth undamaged element stiffness matrix and the ith mode shape of the undamaged or damaged state. Since the location of damage is unknown, the original stiffness matrix is used in the damaged state as an approximation. A damage is assumed to cause a local stiffness reduction affecting the mode shapes in a localized region. Equation (3) shows that with damage occurring in an element of a system, the MSE will change little in the undamaged elements, but there will be a larger change in the damaged elements. Thus, the Modal Strain Energy Change Ratio (MSECR) could be a meaningful indicator for damage localization defined as

$$MSECR_j^i = \frac{|MSE_{ij}^d - MSE_{ij}|}{MSE_{ij}}, \tag{4}$$

where j and i denote the element number and mode number, respectively. If the MSE for several modes are considered together, the $MSECR_j$ of the jth element is defined as the average of the summation of $MSECR_j^i$ for all the modes normalized with respect to the largest value $MSECR_{max}^i$ of each mode.

$$MSECR_j = \frac{1}{m}\sum_{i=1}^{m}\frac{MSECR_j^i}{MSECR_{max}^i}. \tag{5}$$

2.2. VERIFICATION OF THE LOCATION METHOD

The Modal Strain Energy Change Ratio described above can be obtained from experiment, and this section gives some theoretical considerations of the damage indicator.

For a small perturbation in the system described by equation (1)

$$[(\mathbf{K} + \Delta\mathbf{K}) - (\lambda_i + \Delta\lambda_i)\mathbf{M}](\mathbf{\Phi}_i + \Delta\mathbf{\Phi}_i) = 0. \tag{6}$$

Expanding equation (6) to include both the changes in the eigenvalue and the eigenvector,

$$(\mathbf{K} - \lambda_i\mathbf{M})(\mathbf{\Phi}_i + \Delta\mathbf{\Phi}_i) + (\Delta\mathbf{K} - \Delta\lambda_i\mathbf{M})(\mathbf{\Phi}_i + \Delta\mathbf{\Phi}_i) = 0.$$

Neglecting higher order terms, and with $(\mathbf{K} - \lambda_i\mathbf{M})\mathbf{\Phi}_i = 0$, equation (6) leads to

$$(\mathbf{K} - \lambda_i\mathbf{M})\Delta\mathbf{\Phi}_i + (\Delta\mathbf{K} - \Delta\lambda_i\mathbf{M})(\mathbf{\Phi}_i + \Delta\mathbf{\Phi}_i) = 0.$$

Pre-multiplying $\mathbf{\Phi}_r^T$ with $r \neq i$, to both sides of the equation and with $\mathbf{\Phi}_r^T\mathbf{M}\mathbf{\Phi}_i = 0$, equation (6) leads to

$$\mathbf{\Phi}_r^T\Delta\mathbf{K}\mathbf{\Phi}_i + \mathbf{\Phi}_r^T\mathbf{K}\Delta\mathbf{\Phi}_i - \lambda_i\mathbf{\Phi}_r^T\mathbf{M}\Delta\mathbf{\Phi}_i = 0. \tag{7}$$

Since $\mathbf{\Phi}_r^T\mathbf{K} = \lambda_r\mathbf{\Phi}_r^T\mathbf{M}$ for an undamped system, equation (7) can further be rewritten as

$$(\lambda_r - \lambda_i)\mathbf{\Phi}_r^T\mathbf{M}\Delta\mathbf{\Phi}_i = -\mathbf{\Phi}_r^T\Delta\mathbf{K}\mathbf{\Phi}_i. \tag{8}$$

Substituting equation (2c) on $\Delta\mathbf{\Phi}_i$ into equation (8),

$$c_{ir} = \frac{-\mathbf{\Phi}_r^T\Delta\mathbf{K}\mathbf{\Phi}_i}{\lambda_r - \lambda_i}. \tag{9}$$

If $r = i$, the orthogonal relation $\mathbf{\Phi}_i^T\mathbf{M}\mathbf{\Phi}_i = 1$ exists. When $\Delta\mathbf{M} = 0$, it can be shown that $c_{rr} = 0$. This result is the same as Morassi's work [16] in considering the

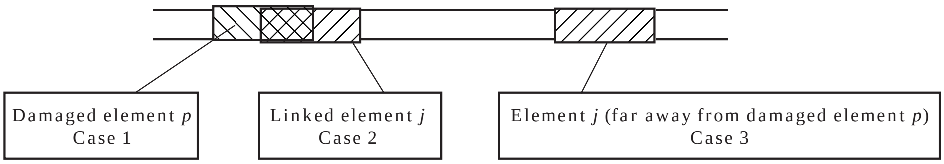


Figure 1. The damaged element p and the target element j.

sensitivity of mode shapes with a crack in a beam element with boundary springs. Therefore, the change in the modal vector can be written as

$$\Delta \mathbf{\Phi}_i = \sum_{r=1}^{m} \frac{-\mathbf{\Phi}_r^T \Delta \mathbf{K} \mathbf{\Phi}_i}{\lambda_r - \lambda_i} \mathbf{\Phi}_r, \quad \text{where} \quad r \neq i. \tag{10}$$

The MSEC after the occurrence of damage is then obtained for the jth element in the ith mode from equation (3) as

$$MSEC_j^i = \mathbf{\Phi}_{di}^T \mathbf{K}_j \mathbf{\Phi}_{di} - \mathbf{\Phi}_i^T \mathbf{K}_j \mathbf{\Phi}_i. \tag{11}$$

Substituting equations (2c) and (10) into equation (11) and neglecting higher order terms, the $MSEC_j^i$ becomes

$$MSEC_j^i = 2\mathbf{\Phi}_i^T \mathbf{K}_j \left(\sum_{r=1}^{m} \frac{-\mathbf{\Phi}_r^T \Delta \mathbf{K} \mathbf{\Phi}_i}{\lambda_r - \lambda_i} \mathbf{\Phi}_r \right), \quad \text{where} \quad r \neq i. \tag{12}$$

Suppose only one damage exists in the structure in member p, substituting equation (2a) into equation (12) gives

$$MSEC_j^i = -2\alpha_p \sum_{r=1}^{m} \frac{1}{\lambda_r - \lambda_i} \mathbf{\Phi}_r^T \mathbf{K}_p \mathbf{\Phi}_i \mathbf{\Phi}_i^T \mathbf{K}_j \mathbf{\Phi}_r, \quad \text{where} \quad r \neq i, \tag{13}$$

where $\mathbf{\Phi}_r^T \mathbf{K}_p$ and $\mathbf{K}_j \mathbf{\Phi}_r$ are two vectors with most of their elements zero except the few elements associated with those DOFs of the elemental stiffness matrix $\mathbf{K}_p$ or $\mathbf{K}_j$.

Equation (13) exhibits the following properties: (1) If $j = p$, i.e., the selected jth element in the FEM carries the damage. The non-zero elements of vectors $\mathbf{\Phi}_r^T \mathbf{K}_p$ and $\mathbf{K}_j \mathbf{\Phi}_r$ correspond to the same DOFs and the value of $MSEC_p$ calculated is much larger than those for the following cases. (2) If $j \neq p$, and the selected jth element of the FEM does not carry the damage. But the element is connected and adjacent to the damaged element p as shown in Figure 1. The sequence of non-zero elements in vector $\mathbf{\Phi}_r^T \mathbf{K}_p$ does not match those of $\mathbf{K}_j \mathbf{\Phi}_r$. But some non-zero

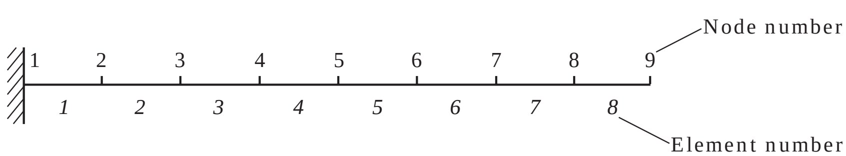


Figure 2. An eight-element cantilever beam model.

TABLE 1

Modal strain energy change in a simulated damaged cantilever beam

		$MSEC_j^i$					$MSEC_j$
Damage cases	Element no.	from 1st mode	from 2nd mode	from 3rd mode	from 4th mode	from 5th mode	from five modes
Damage in element 1	1	1	1	1	1	0·304*	0·861†
	2	−0·271	−0·335	−0·466	−0·191	1	0·453
	3	−0·253	−0·388	−0·063	0·574	−0·855	0·427
	4	−0·208	−0·161	0·150	−0·714	0·170	0·281
	5	−0·147	−0·090	−0·046	−0·319	−0·142	0·149
	6	−0·084	−0·020	−0·294	0·212	−0·296	0·181
	7	−0·033	−0·006	−0·243	−0·416	0·028	0·145
	8	−0·004	0·0002	−0·038	−0·145	−0·208	0·079
Damage in element 2	1	−0·294	−0·557	−0·973	−1	1	0·765
	2	1	1	1	−0·133*	0·980	0·823†
	3	−0·244	−0·082	−0·056	0·208	−0·749	0·268
	4	−0·202	−0·269	0·176	0·216	0·059	0·184
	5	−0·143	−0·050	0·092	0·003	−0·518	0·161
	6	−0·082	−0·042	−0·099	0·129	−0·150	0·073
	7	−0·032	0·002	−0·120	0·256	−0·355	0·153
	8	−0·004	−0·001	−0·021	0·054	−0·267	0·069
Damage in element 3	1	−0·312	−0·830	−1	0·314	−0·471	0·585
	2	−0·277	−0·106	−0·427	0·022	−0·412	0·249
	3	1	1	0·241*	1	1	0·848†
	4	−0·179	0·051	0·130	−0·564	0·144	0·214
	5	−0·127	−0·106	0·360	−0·159	−0·005	0·151
	6	−0·073	−0·0002	0·419	−0·027	−0·240	0·152
	7	−0·028	−0·010	0·242	−0·460	0·114	0·171
	8	−0·003	0·001	0·033	−0·125	−0·129	0·058

* Incorrect identification.
† Correct identification.

elements in both vectors $\mathbf{\Phi}_r^T\mathbf{K}_p$ and $\mathbf{K}_j\mathbf{\Phi}_r$ correspond to the same DOFs. So the value of the $MSEC_j$ calculated is smaller than that of $MSEC_p$ but larger than that for the following case. (3) If $j \neq p$, i.e., the selected jth element in the FEM does not carry the damage, and the element j is far away from the damaged element p. The DOFs corresponds to the non-zero elements in vector $\mathbf{\Phi}_r^T\mathbf{K}_p$ does not match with those in vector $\mathbf{K}_j\mathbf{\Phi}_r$. The value of $MSEC_j$ calculated from equation (3) is therefore very small.

A numerical example of a cantilever beam consisting of eight elements, as shown in Figure 2, is used to illustrate these properties. A reduction of stiffness of 10% is assumed to occur in elements 1, 2 and 3 in turn, and the Modal Strain Energy Changes are calculated for the first five modes and the values are shown in Table 1. Ten analytical modes are included in the calculation in equation (13). The Modal Strain Energy of each element has been normalized with respect to the largest value of the same group and the $MSEC_j$ of an element is the average of the absolute value of the Modal Strain Energy Change for all the modes.

The Modal Strain Energy Change has the largest value at the damaged element and a smaller value at the adjacent-to-damage elements. The value is very small at elements far away from the damaged elements. However, elements corresponding to nodal points of the mode shape have exceptionally large and small values, and this may sometimes give the wrong indication of the damage location, e.g., incorrect identification of damage from the 5th mode for element 1; from the 4th mode for element 2; and from the 3rd mode for element 1. This disadvantage of using one single mode can be overcome by using the Modal Strain Energy Change for several modes as those values shown in Table 1 with an † indicating clearly the damage location.

The Modal Strain Energy Change in higher modes do not contribute much in the identification of the damage location. This can be explained by: (a) the size of discretized element of the model is of the same order as the nodal point/line density of the structure; and (b) the effect of truncation of analytical vibration modes above the mth mode is smaller for the higher modes, as seen from equation (13).

The above discussions give some evidence to support the intuitive belief that measured mode shapes are not sensitive to local change in stiffness except when the measurement is made in or close to the damage domain.

The Modal Strain Energy Change of a damage element is larger than that of any other undamaged element. Elements that are linked with the damaged one have a smaller Modal Strain Energy Change value. And if an element is far away from the damage element, the Modal Strain Energy Change of this element will be much smaller. Similar results can also be obtained when there are multiple damages in the structure. The Modal Strain Energy Change values from the lower modes give clear indication of the damage location and the use of combined *MSEC* values from several modes is recommended.

3. EXAMPLES

To evaluate the performance and robustness of the Modal Strain Energy Change Ratio in locating damage, two simulated problems are investigated. first example makes use of the widely-used European Space Agency truss structure. Several simulated damage cases are investigated. The purpose of the simulation is to assess the effectiveness of this approach when the measured modes are incomplete with or without noise. The second example studies the effect of systematic error and random error on the proposed damage localization approach in a two-storey steel plane frame structure.

3.1. SIMULATED DAMAGE LOCALIZATION

The two-dimensional truss structure is shown in Figure 3. The finite element model of the structure consists of 78 two-dimensional beam elements, 74 nodes and 222 DOFs. Each node has three DOFs with motion confined in the plane of the structure. The numbers given in the figure are element numbers. The geometrical and physical data of the structure used in the initial FEM are shown in Figure 3.

Two damage cases are assumed: (a) Single damage in the 25th element in the lower chord with the stiffness decreased by 15%; and (b) Multiple damages in the 25th element in the lower chord, in the 53rd element in the inclined member and in the 70th element in the vertical member with their stiffnesses decreased by 15%.

The first five modes of vibration are measured in the following damage localization procedure. The analytical mode shapes are taken as the undamaged mode shapes. These mode shapes are contaminated with 5% random noise as the "measured" mode shapes. Figures 4 to 11 shown the damage localization results for each case. The following symbols in the legend are used in the figures: n denotes complete measured modes with noise; c denotes complete measured modes without noise; i denoted incomplete measured modes without noise; and $n - i$ denotes incomplete measured modes with noise.

Figures 4 to 7 show the results of damage location by using equation (4) or (5) with complete "measured" vibration modes. The x-axis gives the element number and the y-axis is the value of Modal Strain Energy Change Ratio for each element. Figures 4 and 6 show the damage location results detected by using only one mode. The results from using five modes with and without noise are compared in Figures 5 and 7. It is seen that both cases of a single damage and multiple damages can be identified by using a single mode or multiple modes. The value of $MSECR$ for the damaged element is decreased by 20% due to the presence of measurement noise. However this effect is smaller when multiple modes are used.

Figures 8 and 9 show that the damages can also be located when the "measured" vibration modes are incomplete. Only 78 translational DOFs are "measured" which is about 35% of the total DOFs of the FEM of the structure. They are so selected such that information collected is relatively uniform from all parts of the structure, and information from the rotational DOFs are not considered. The measured points are shown in Figure 3 as black dots. The incomplete "measured" modes are expanded by using a new modal expansion method described in Shi

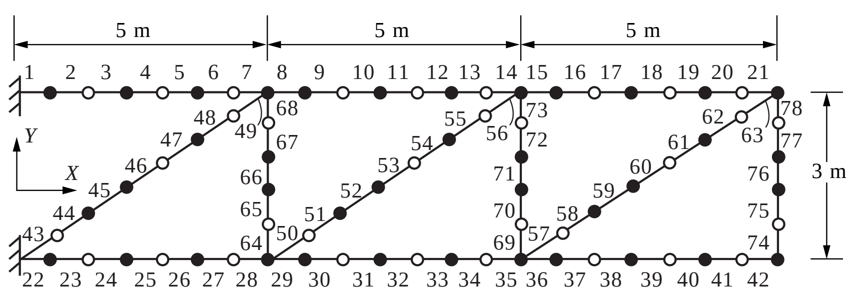


Figure 3. The European Space Agency structure and the measurement points. ●, Measured point (x- and y-direction) in the incomplete "measurement". Finite Element Model: $E = 0{\cdot}75 \times 10^{11}$ Pa, $I = 0{\cdot}0756$ m^4, $\rho = 2800$ kg/m^3; vertical elements: $A = 0{\cdot}6 \times 10^{-2}$ m^2; diagonal elements: $A = 0{\cdot}3 \times 10^{-2}$ m^2; horizontal elements: $A = 0{\cdot}4 \times 10^{-2}$ m^2.

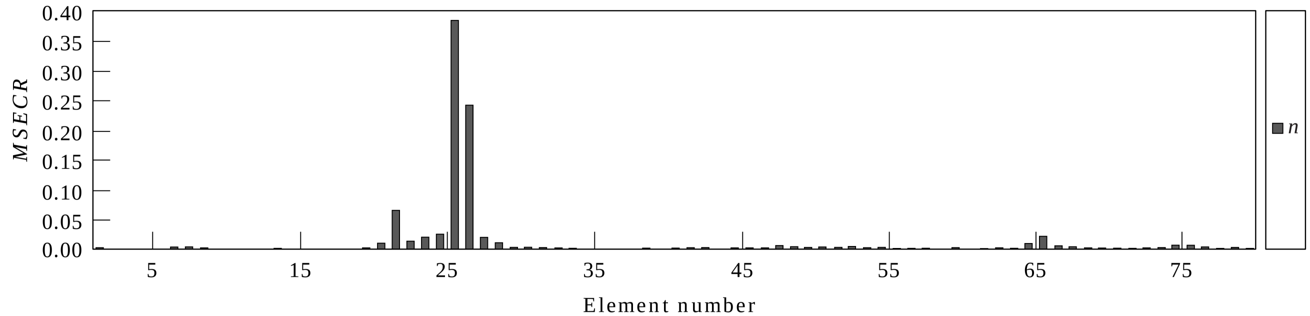


Figure 4. Damage in 25th (15%) element (fourth mode: 5% noise).

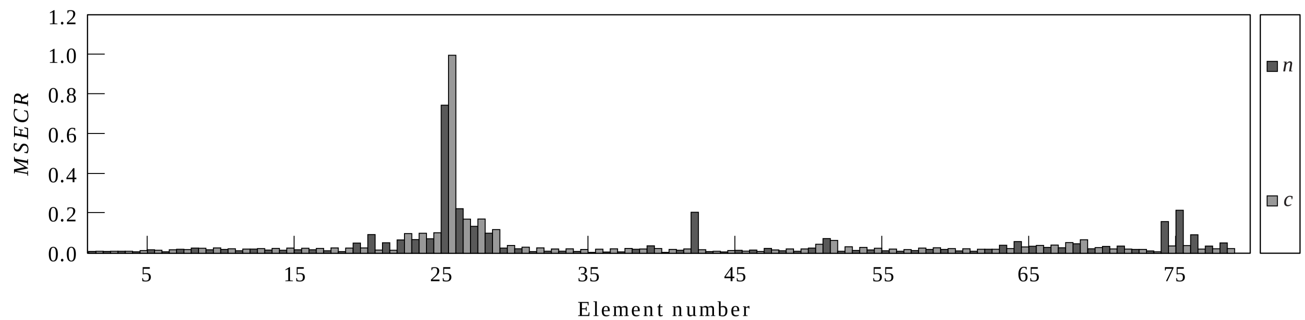


Figure 5. Damage in 25th (15%) element (five modes: 5% noise).

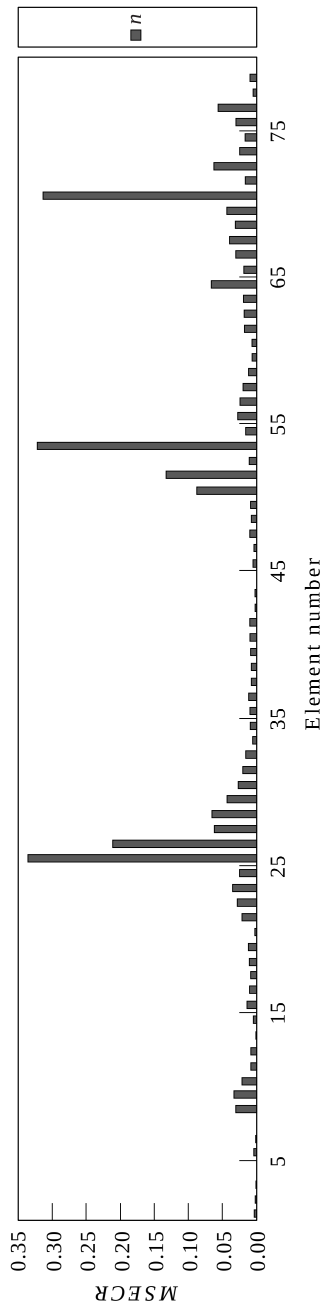


Figure 6. Damages in 25th, 53rd and 70th (15%) element (third mode: 5% noise).

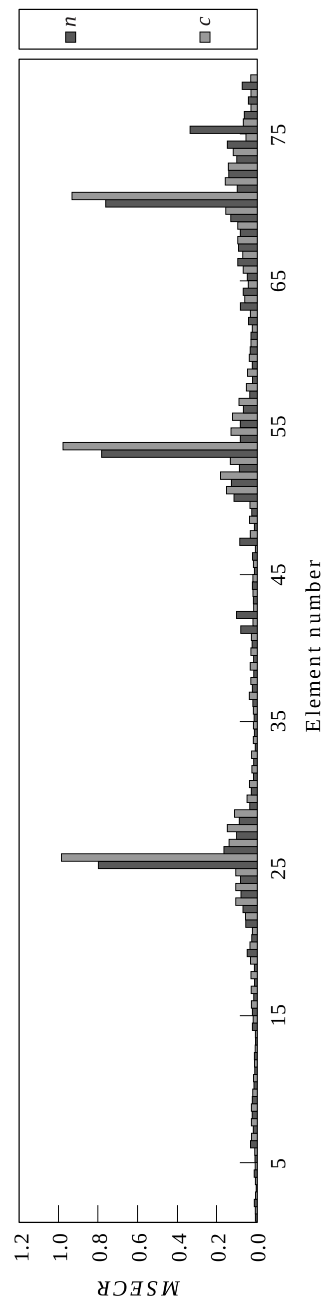


Figure 7. Damages in 25th, 53rd and 70th (15%) element (five modes: 5% noise).

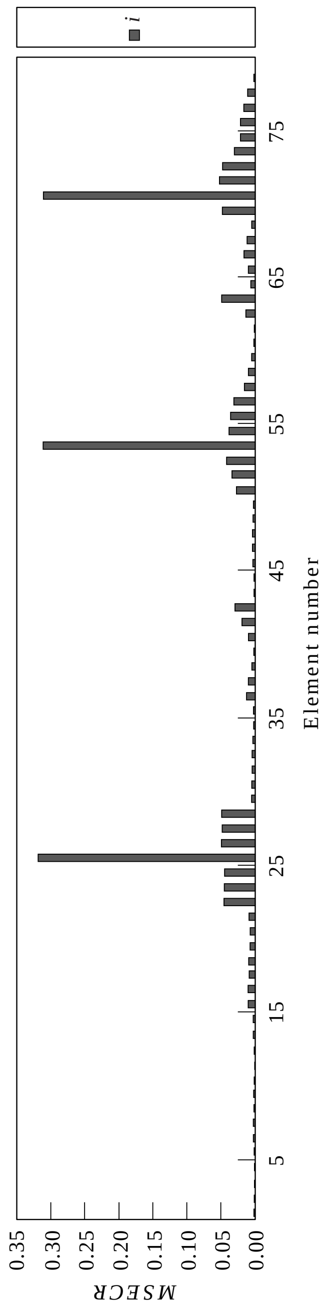


Figure 8. Damages in 25th, 53rd and 70th (15%) element (incomplete second mode).

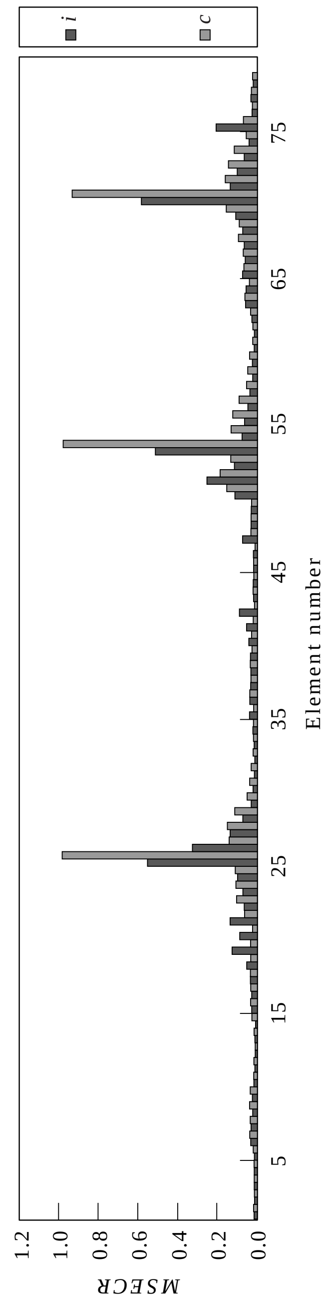


Figure 9. Damages in 25th, 53rd and 70th (15%) element (incomplete first five modes).

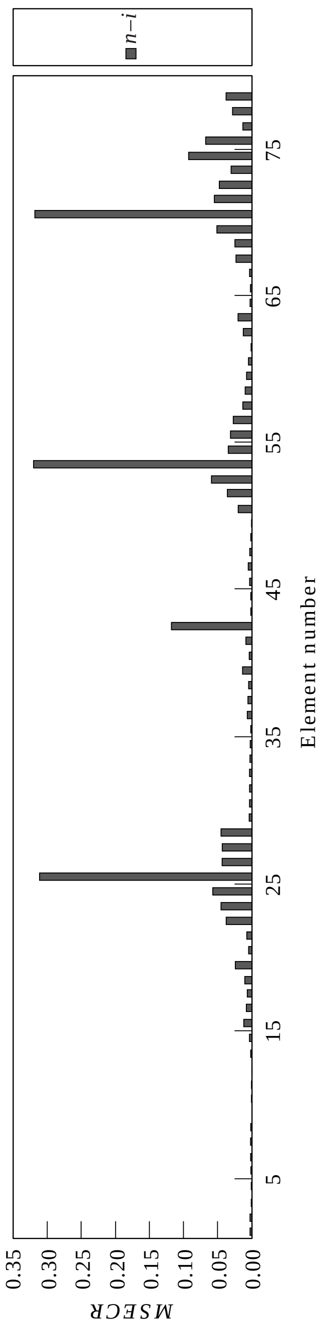


Figure 10. Damages in 25th, 53rd and 70th (15%) element (incomplete second mode with 5% noise).

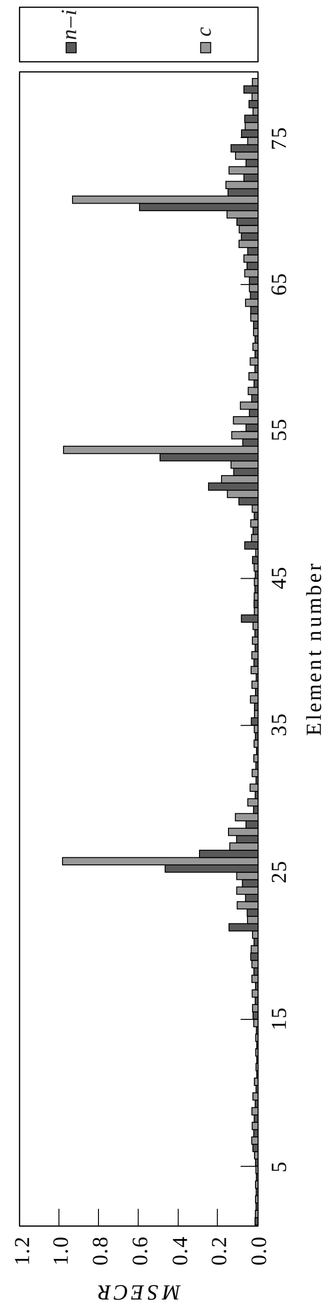
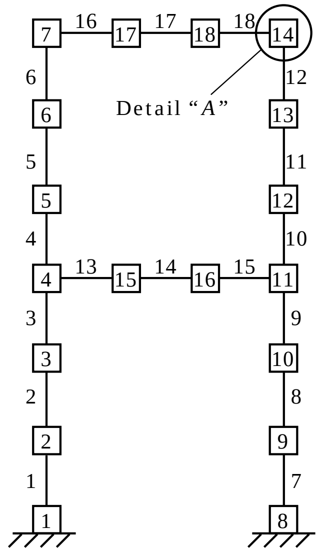
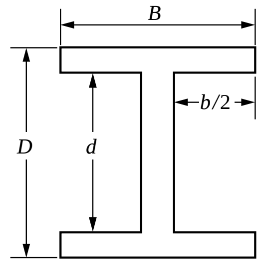


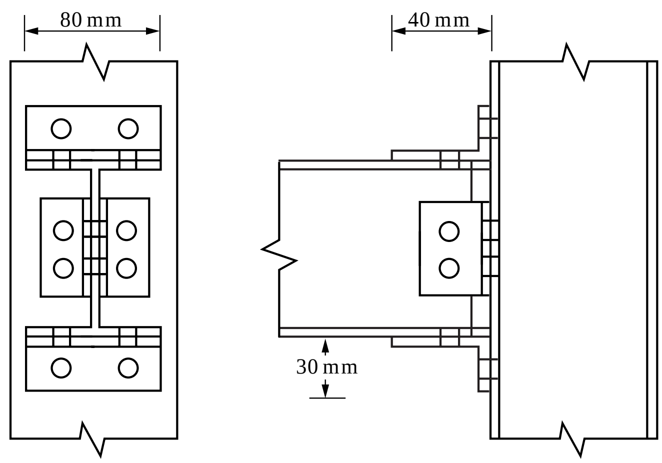
Figure 11. Damages in 25th, 53rd and 70th (15%) element (incomplete first five modes with 5% noise).





	Column	Beam
B	0.0688	0.0761
D	0.1	0.151
b	0.0638	0.0711
d	0.085	0.1359

Units: metres



Detail "*A*"

Figure 12. Finite element model of the two-story frame structure. $E = 2{\cdot}1 \times 10^{11}\ \mathrm{N\ m^{-2}}$, $\rho = 7800\ \mathrm{kg/m^3}$.

et al. [17] to the full dimension of the FEM. The expanded modes are then used in the damage localization. Figure 8 gives the result of multiple damages identification using only the second mode without noise. The multiple damages located using five modes are shown in Figure 9. It is seen that the incompleteness of the "measured" mode greatly affects the damage identification result. The value of *MSECR* is reduced by about 45% compared with those computed from complete modes.

The results of multiple damages detected from incomplete and noisy modes are shown in Figures 10 and 11. The combined effect of incompleteness and measurement noise reduces the *MSECR* value by about 50% when compared with values obtained from the complete mode.

TABLE 2

MAC—with damage at node 4 of the frame structure

Mode: damaged	Undamaged Frequency (Hz)	1 22·49	2 74·17	3 198·51	4 221·77	5 261·32	6 280·17
1	19·33	0·9982	0·0176	0·0001	0·0001	0·0114	0·0085
2	73·75	0·0074	0·9987	0·0007	0·0003	0·0267	0·0013
3	195·25	0·0000	0·0002	0·9700	0·0588	0·0166	0·0017
4	221·02	0·0001	0·0000	0·0077	0·9759	0·0128	0·1115
5	241·26	0·0235	0·0330	0·0118	0·0040	0·9397	0·0054
6	275·87	0·0043	0·0069	0·0069	0·1043	0·0012	0·9870

TABLE 3

MAC—with damage at node 7 of the frame structure

Mode: damaged	Undamaged Frequency (Hz)	1 22·49	2 74·17	3 198·51	4 221·77	5 261·32	6 280·17
1	21·83	0·9979	0·0295	0·0019	0·0023	0·0325	0·0034
2	67·50	0·0053	0·9834	0·0047	0·0019	0·0340	0·0002
3	193·30	0·0004	0·0022	0·9535	0·0002	0·0377	0·0006
4	219·83	0·0001	0·0015	0·0689	0·9173	0·0221	0·0103
5	247·09	0·0145	0·0131	0·0116	0·0598	0·8359	0·1291
6	278·24	0·0000	0·0001	0·0006	0·0017	0·1021	0·7513

TABLE 4

MAC—with damages at nodes 7 and 11 of the frame structure

Mode: damaged	Undamaged Frequency (Hz)	1 22·49	2 74·17	3 198·51	4 221·77	5 261·32	6 280·17
1	18·67	0·9946	0·0373	0·0006	0·0082	0·0206	0·0011
2	67·10	0·0024	0·9796	0·0030	0·0106	0·0032	0·0002
3	191·99	0·0007	0·0032	0·9788	0·0018	0·0115	0·0049
4	217·08	0·0001	0·0001	0·0513	0·7016	0·1336	0·0702
5	235·02	0·0203	0·0153	0·0001	0·2766	0·6690	0·0910
6	273·42	0·0050	0·0017	0·0126	0·0019	0·1182	0·5678

Results suggest that the Modal Strain Energy Change Ratio can effectively be used to locate the structural damage in a single element and in multiple elements with incomplete and noisy "measured" modes.

3.2. EXPERIMENTAL DAMAGE LOCATION

A two-storey steel plane frame structure (2·82 m high and 1·41 m wide), shown in Figure 12, is used in the experimental damage localization in the laboratory.

The sturcture is modelled by 18 two-dimensional beam elements of equal length. The beams are connected to the column horizontally by top- and seat-angles and double web-angles with bolts and nuts. Details of this type of beam–column connection are also shown in Figure 12. The initial rotational stiffness of this kind of beam–column connection is approximately $3{\cdot}0 \times 10^6$ Nm/rad from static tests [18]. Details of the geometrical and physical information and the FEM of the structure are shown in Figure 12. The bottom of the columns are fully welded to base plates which are welded to the rigid floor, and the same rotational stiffness as for the beam–column connection is assumed for these supports in the finite element modelling.

Three damage cases on the frame structure are studied: (1) damage at node 4; (2) damage at node 7; and (3) damages at nodes 7 and 11.

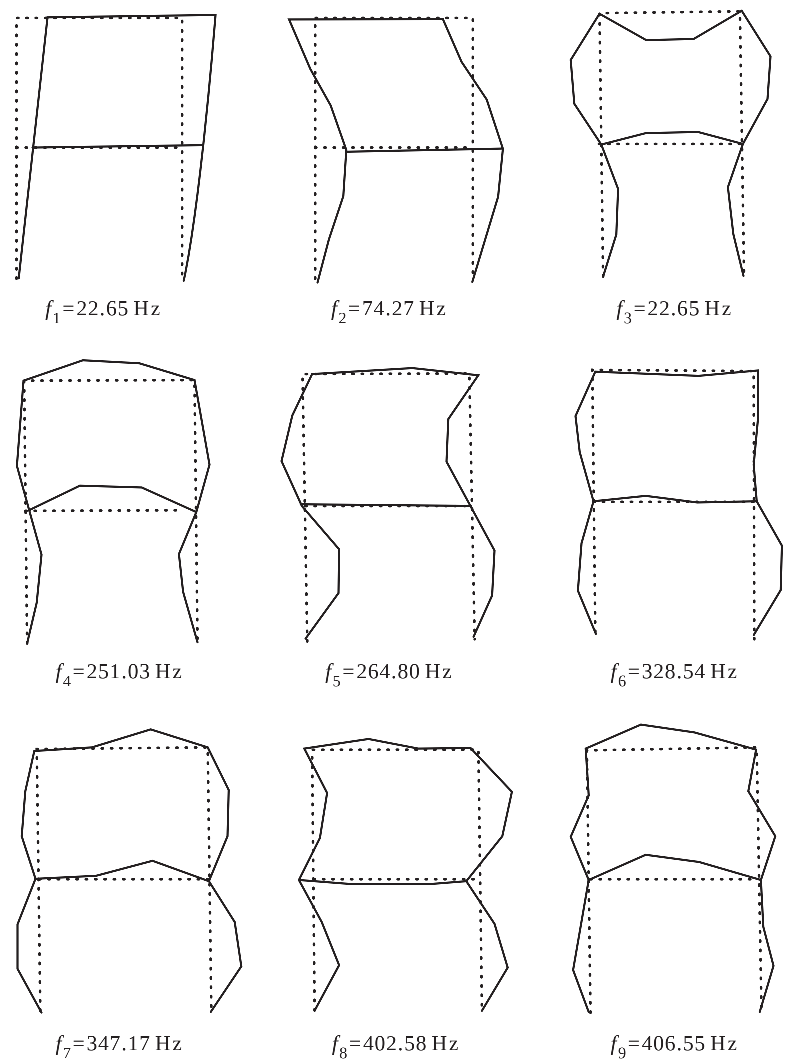


Figure 13. Mode shapes of the frame structure.

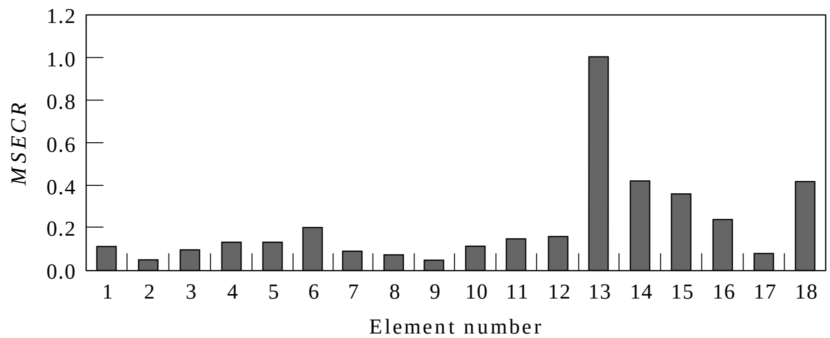


Figure 14. Damage at node 4 (element 13) (the first mode).

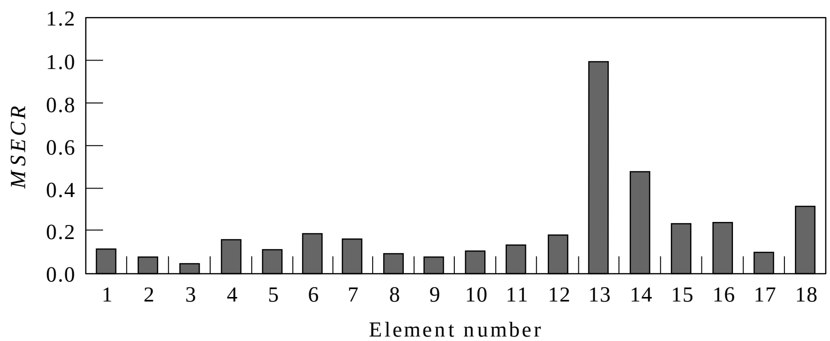


Figure 15. Damage at node 4 (element 13) (the second mode).

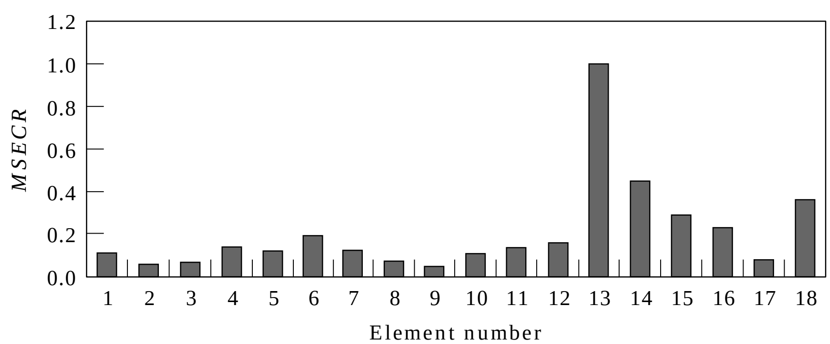


Figure 16. Damage at node 4 (element 13) (two modes).

The damage is simulated by removing both the top- and seat-angles at the joint. This release of restraints to the joints will affect only the stiffness of the horizontal member but not the vertical member. Nine modal frequencies and mode shapes of the frame before and after the "damage" is introduced are measured [19] and determined using the Structural Modal Analysis Package [20]. The modal frequencies are shown in Table 2 to 4 and the theoretical mode shapes are shown in Figure 13. Only the horizontal translational DOFs at nodes 2 to 7 and 9 to 14, and the vertical translational DOFs at nodes 15 to 18 are measured. The incomplete mode shapes are then expanded with the modal expansion method [17].

The accuracy of the FEM of the structure is checked by the Modal Assurance Criteria (MAC) [21] between the analytical mode shapes and the measured mode shapes. The MAC values are larger than 0·98 for the first two modes, as seen in Table 2 to 4. Therefore, the FEM is considered accurate enough to represent the structure for illustration of this method, and it is directly used as a reference model without any system updating. The first two modes are used in the damage localization procedure.

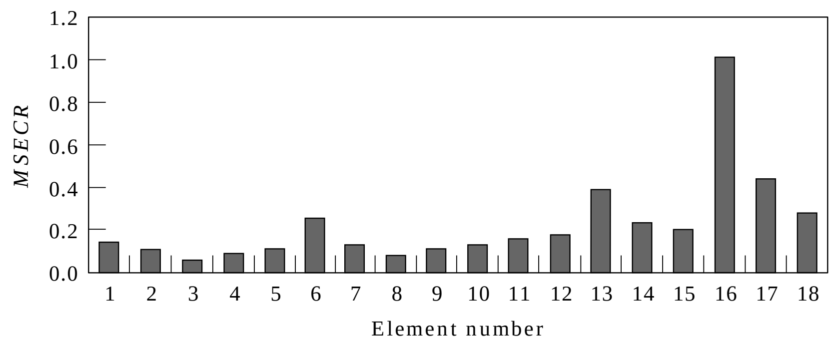


Figure 17. Damage at node 7 (element 16) (the first mode).

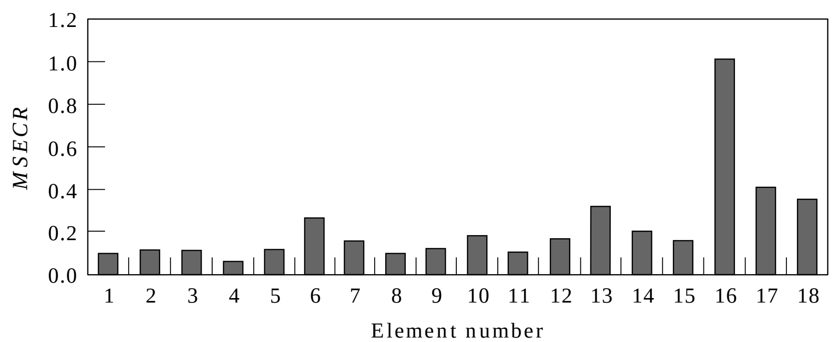


Figure 18. Damage at node 7 (element 16) (the second mode).

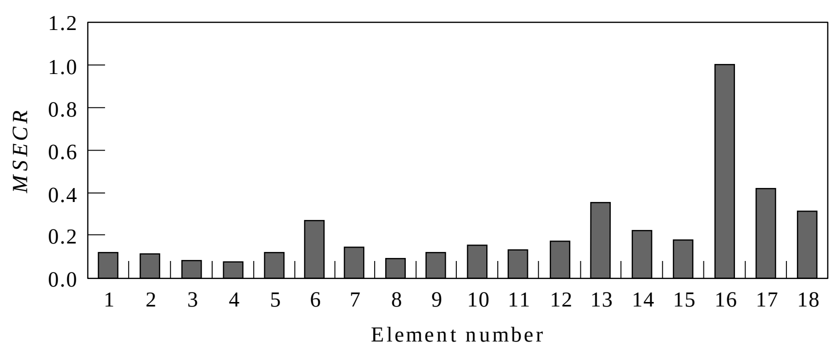


Figure 19. Damage at node 7 (element 16) (two modes).

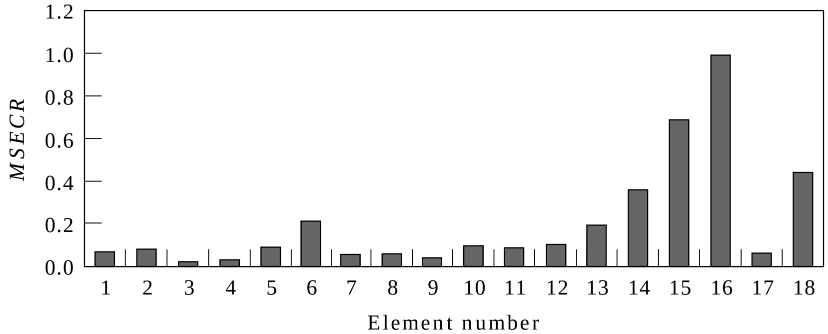


Figure 20. Damages at nodes 7 and 11 (elements 15 and 16) (the first mode).

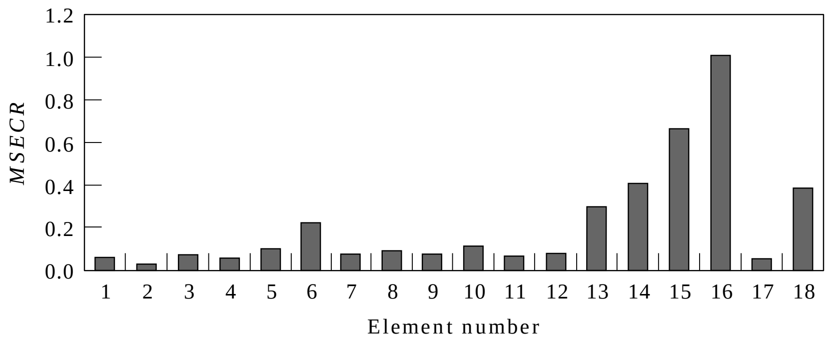


Figure 21. Damages at nodes 7 and 11 (elements 15 and 16) (the second mode).

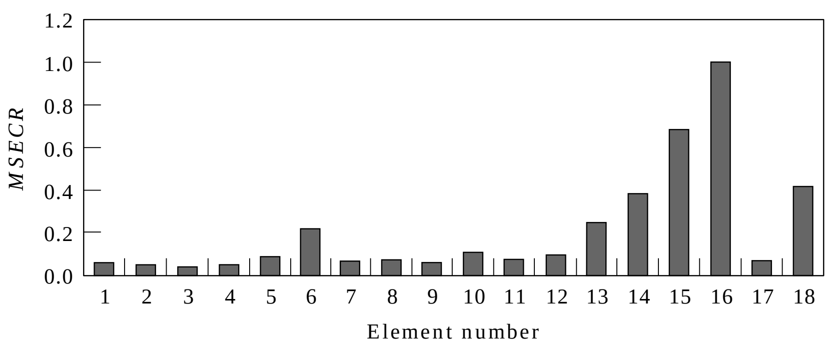


Figure 22. Damages at nodes 7 and 11 (elements 15 and 16) (two modes).

The damage location is successfully detected by using equation (4) or (5). The results are shown in Figures 14 to 22. Figures 14 to 16 show damage exists in element 13 which has one end connected to the column at node 4 by using only the first mode, the second mode and the first two modes, respectively. Damage at node 7 is identified as damage in element 16 with one end connected to the column at node 7, and the results are shown in Figures 17 to 19. Multiple damages at nodes 7 and 11 are located as damages in elements 15 and 16 with one end connected to the column at nodes 11 and 7, respectively, and the results are shown in Figures 20 to 22. The normalized *MSECR* in the damaged element is 1·0, and the values of other elements are less than 0·48 when there is a single damage in the structure. Figures 20 to 22 show that the damage at node 7 is much easier to locate than damage at node 11. The normalized *MSECR* in the elements 16 and 15 are 1·0 and close to 0·7, respectively. The values of the other elements are less than 0·45 when two damages exist in the structure.

4. CONCLUSIONS

A damage localization method using Modal Strain Energy Change has been presented. The computation of this change involves only the elemental stiffness matrix and the analytical mode shpaes of the structure, and it is sensitive to the existence of damage. No other *a priori* information about the structure is required. A localization procedure is illustrated by several simulated damage cases in the European Space Agency truss structure and by the experimental damage localization studies in a two-dimensional frame structure using complete and incomplete measurements. Results indicate that this method is effective and robust to locate single or multiple damages in the structure. Measurement noise and incompleteness of measured modes greatly affect the damage location result. However, good results can still be obtained using results from more than one measured mode.

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